MYP Mathematics

A concept-based approach

4&5
Extended

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E1.1

What if we all had eight fingers?

Global context: Scientific and technical innovation

FORM

Objectives

- Understanding the concept of a number system
- Counting in different bases
- Converting numbers from one base to another
- Using operations in different bases

Inquiry questions

F
- How have numbers been written in history?
- What is a number base?
- How can you write numbers in other bases?

C
- How are mathematical operations in other bases similar to and different from operations in base 10?

D
- Would you be better off counting in base 2?
- How does form influence function?

ATL **Communication**

Use intercultural understanding to interpret communication

You should already know how to:

• use the operations of addition, subtraction and long multiplication in base 10, without a calculator	**1** Calculate these by hand: **a** 10 442 + 762 **b** 10 887 − 7891 **c** 27 × 43 **d** 14 078 × 71
• understand place value	**2** Write down the value of the 5 in: **a** 351 **b** 511 002 **c** 15 **d** 1.5

F Numbers in different bases

- How have numbers been written in history?
- What is a number base?
- How can you write numbers in other bases?

Humans have used many different ways to record numbers. How would you write down the number of green bugs in this diagram?

You might have written a symbol 5, or the word “five”. You could have used a word in a different language, or maybe even a tally: 𝍸. Each of these represents the number in a different way, but they all represent the same quantity.

Different cultures use different forms of notation to represent number. The ancient Egyptian hieroglyphic number system was an **additive system**. Each symbol has a different value and you find the total value of the number by adding the values of all the symbols together.

Egyptian numerals use these symbols:

 stroke	 heelbone	 coiled rope
1	10	100

 lotus flower	 pointed finger	 tadpole	 scribe
1000	10 000	100 000	1 000 000

Green shield bugs are sometimes called green stink bugs, as they produce a pungent odor if handled or disturbed.

The number 11 is written I∩, and 36 is written IIIIII∩∩∩.

ATL

Reflect and discuss 1

- Write each number in Egyptian numerals:

5 | 32 | 126 | 99 | 100 | 10 240

- Write these numbers as ordinary (decimal) numbers:

- What are the advantages of the Egyptian number system?
- What are the disadvantages?

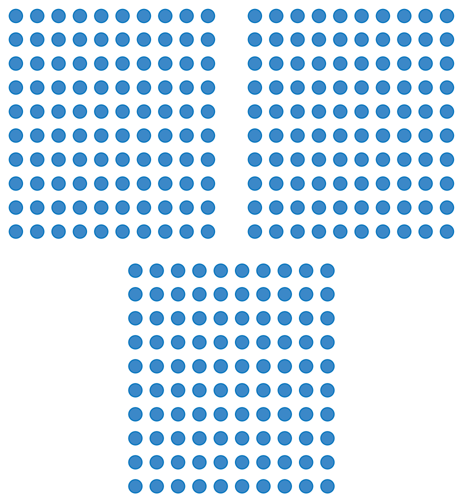
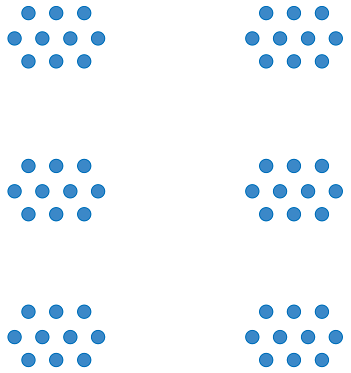
> Our number system, often called Arabic numerals, is a **place value system**. The position of a digit tells you its value.

The two numbers below have the same four digits, but digits do not always represent the same amount.

3642 and 4623
↑ ↑
3 thousands 3 units

A digit in the furthest right column represents individual objects: *units*.
Moving left, the second column represents collections of ten units: *the tens column*.
The third column represents collections of ten tens: *the hundreds column*.
The pattern continues: each column is worth 10 times more than the column to its right.

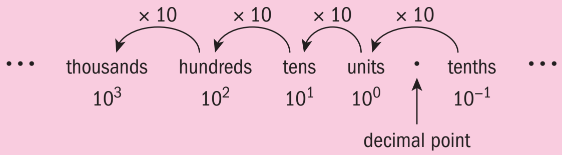
The number 364 actually represents a collection of that many objects:

hundreds	tens	units
3	6	4
		

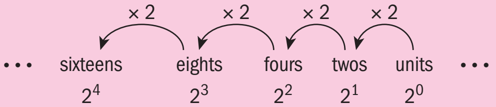
Reflect and discuss 2

- Does the symbol 0 have different values in the numbers 205 and 2051? Does it represent something different?
- The Roman numerals do not have a symbol to represent zero, so why does the place value system need a symbol for zero?

Our number system is known as decimal (also as base ten, or denary) because the value of each place value column is ten times the value of the column to its right.



You can make a place value system with any natural number base.
In binary (base two) each column is worth twice the column to its right.



The binary number $10\,111_2$ represents a set containing:

sixteens	eights	fours	twos	units
1	0	1	1	1
●●●● ●●●● ●●●● ●●●●	○○○○ ○○○○	●●●●	●●	●

The subscript 2 means that the number $10\,111_2$ is written in base 2.

Computer logical systems use base 2. This is why you see powers of 2 in lots of contexts relating to computers. For example, SD cards which store 8 GB (gigabytes), 16 GB, 32 GB, 64 GB and so on, rather than 10 GB, 20 GB, etc. Also, whereas the prefix *kilo* usually means 1000 (e.g. there are 1000 meters in a kilometer), in computing, the term *kilo* means 1024 (2^{10}), so there are 1024 bytes in a kilobyte.

Tip

Base 3 - ternary, or trinary

Base 4 - quaternary

Base 8 - octal

Base 12 - duodecimal

Base 16 - hexadecimal (widely used in computing)

The base of a number system tells you how many unique symbols the number system has. Base 10 has ten unique number symbols, 0 through 9. Base 2 has two unique symbols, 0 and 1.

Example 1

Find the value of $12\,011_3$ in base 10.

3^4	3^3	3^2	3^1	3^0
81	27	9	3	1
1	2	0	1	1

Write the powers of 3 above the digits, starting with $3^0 = 1$ at the right-hand side.

$81 + 2 \times 27 + 3 + 1 = 139$

Add up the parts that make up the number.

$12\,011_3 = 139_{10}$

Use subscripts to show the base.

Practice 1

1 Find the value of each binary number in base 10.

a $10\,111_2$ **b** $11\,001_2$ **c** $1\,101\,101_2$

2 Convert each number to base 10.

a $21\,002_3$ **b** $22\,101_3$ **c** 412_5

d 332_5 **e** 64_8 **f** 77_9

Problem solving

3 Write these numbers in ascending order:

1010_4 $100\,011_2$ 1011_3 1111_5

4 The number $1\,101\,011\,000_2 = 856_{10}$.

a Describe the relationship between $1\,101\,011\,000_2$ and $110\,101\,100_2$.

b Find the value of $1\,101\,011_2$.

> Music is the pleasure the human mind experiences from counting, without being aware that it is counting.
>
> – *Gottfried Leibniz*

Exploration 1

1 Use this table to explore questions **a** to **g**.

3^7	3^6	3^5	3^4	3^3	3^2	3^1	3^0
2187	729	243	81	27	9	3	1

a Explain how the value of 3^7 tells you that the number 1038_{10} will have 7 digits in base 3.

b $1038_{10} = 729_{10} + 309_{10}$.

Explain how this sum tells you that the first digit of 1038_{10} in base 3 will be 1.

▶ Continued on next page

c $309_{10} = 243_{10} + 66_{10}$.

Explain how this sum tells you that the second digit of 1038_{10} in base 3 will also be 1.

d $66_{10} < 81_{10}$.

Explain how this inequality tells you that the third digit of 1038_{10} in base 3 will be 0.

e $66_{10} = 2 \times 27_{10} + 12$.

Explain how this sum tells you that the fourth digit of 1038_{10} in base 3 will be 2.

f Continue the process to find the value of 1038_{10} in base 3.

g Check your solution by converting it back into base 10.

2 Use the method in step **1** to convert:

a 1000_{10} to base 2 **b** 513_{10} to base 3 **c** 673_{10} to base 4.

3 The following algorithm produces the digits of a number n in base b, starting with the units digit and then working to the left.

- Start with the number, n. Divide n by b and find the quotient q, and the remainder r.
- Write down the value of r.
- If $q > 0$, replace n with the value of q; otherwise stop.
- Repeat from the beginning with the new value of n, and record new remainders to the left of any you have already written down.

4 Here the algorithm is used to convert 1038_{10} into base 3.

	$n \div b$	
n	q	r
1038	346	0
346	115	1
115	38	1
38	12	2
12	4	0
4	1	1
1	0	1
	Read the number in base b upwards from here: 1 102 110	

$1038 \div 3 = 346\ r\ 0$. The units digit is 0. Change n to 346.

$346 \div 3 = 115\ r\ 1$. The 3s digit is 1. Change n to 115.

$115 \div 3 = 38\ r\ 1$. The 3^2 digit is 1. Change n to 38.

$38 \div 3 = 12\ r\ 2$. The 3^3 digit is 2.

$12 \div 3 = 4\ r\ 0$. The 3^4 digit is 0.

$4 \div 3 = 1\ r\ 1$. The 3^5 digit is 1.

$1 \div 3 = 0\ r\ 3$. The 3^6 digit is 0. Stop because $q = 0$.

Use the algorithm to convert:

a 1000_{10} to base 2 **b** 513_{10} to base 3 **c** 673_{10} to base 4.

Practice 2

1 Convert 999_{10} to:

a base 2 **b** base 3 **c** base 4 **d** base 5

Use the method you prefer to convert numbers to different bases.

2 a Find the value of 472_8 in base 10.

b Hence find the value of 472_8 in base 5.

3 By first converting to base 10, find the value of these numbers in the given base.

a 223_5 in base 7 **b** 431_8 in base 2 **c** 214_6 in base 2

d 1011_2 in base 6 **e** $110\,111_2$ in base 8 **f** $110\,213_4$ in base 9

g 8868_9 in base 3 **h** $101\,101_2$ in base 4 **i** 2468_9 in base 8

Problem solving

4 Four students write the same number in different bases:

Alberto: 1331_a Benito: 2061_b Claudio: 3213_c Donatello: 1000_d

a Determine which of the four students used the largest base. Explain how you know.

b Use your answer to part **a**, and any other information you can gain from the students' numbers, to list the numbers a, b, c and d in ascending order.

c Determine the minimum possible value for b.

d Find values of a, b, c and d such that $1331_a = 2061_b = 3213_c = 1000_d$.

e Use your answer to **d** to find the value of the number in base 10.

Exploration 2

A bottle factory packs 12 bottles to a box. There are 12 boxes in a crate.

A shipping container will hold 12 crates.

1 Determine the number of bottles in each crate.
Determine the number of bottles in each shipping container.

2 A customer orders 600 bottles. Find the number of crates and boxes to fulfil this order.

3 A customer orders 81 bottles. Determine the number of complete boxes and single bottles for this order.

4 The table on page 9 shows four different orders, with some information missing. Calculate appropriate values for the shaded cells.
The basic price per bottle is €1.40.

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Customer name	Total ordered	Notes	Containers	Crates	Boxes	Singles	Total cost
Mr Antinoro	**a**	n/a	0	5	0	0	**b**
Mr Drouhin	6000	10% discount on complete containers	**c**	**d**	**e**	**f**	**g**
Herr Müller	**h**	**i** (% discount on the whole order)	4	4	1	0	€8400
Mrs Symington	1500	n/a	**j**	**k**	**l**	**m**	**n**

5 An employee suggests that since all orders are made of a number of containers, crates, boxes and singles, the company does not need to repeat the headings every time, so an order of 3 crates, 7 boxes and 0 singles could be written as 370.

- **a** Write Mr Antinoro's, Mr Drouhin's and Herr Müller's orders using this convention.
- **b** Explain how notating the orders in this way relates to writing numbers in non-denary number bases.
- **c** Describe the problem in writing Mrs Symington's order in this way.

ATL

Reflect and discuss 3

- Why do you think people most commonly use base 10 to count?
- When is the number 12 commonly used as a base? What makes the number 12 a convenient number to use?

Base 10 (decimal) uses ten different symbols to describe the whole numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Similarly, base 2 uses two symbols: 0 and 1. Base 12 (duodecimal) requires twelve symbols, but you cannot use '10' or '11' because these both involve two digits.

When you write numbers in bases greater than 10, letters are used for the extra symbols needed.

In base 12, the symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A and B.

Sometimes lowercase letters a and b are used instead of uppercase A and B.

Example 2

Find the value of $A3B_{12}$ in base 10.

12^2	12^1	12^0
144	12	1
A	3	B

The letter A represents 10, and B represents 11.

$$A3B_{12} = (10 \times 144 + 3 \times 12 + 11 \times 1)_{10}$$
$$= 1487_{10}$$

Example 3

Find the value of 500 in base 16.

	$n \div b$	
n	q	r
500	31	4
31	1	F
1	0	1

$500_{10} = 1F4_{16}$

$500 \div 16 = 31$ r 4. The units digit is 4. Change n to 31.

$31 \div 16 = 1$ r 15. The symbol for 15 is F. Change n to 1.

$1 \div 16 = 0$ r 1. The 16^2 digit is 1. Stop because $q = 0$.

Practice 3

1 Convert to base 10:

a 210_{12} **b** 301_{16} **c** BAA_{12} **d** $G0_{20}$

2 Find the value in base 16:

a 190_{10} **b** 2766_{10} **c** $47\,806_{10}$

d $48\,879_{10}$ **e** $51\,966_{10}$ **f** $64\,206_{10}$

Problem solving

3 Hexadecimal (base 16) codes are often used as passwords for home Wi-Fi hubs and other digital services.

a Explain why there are 256_{10} possible 2-character passwords in hexadecimal.

b Find the number of possible 10-character hexadecimal passcodes. Give your answer to a suitable degree of accuracy.

Counting in 20s

US President Abraham Lincoln's "Gettysburg Address" began:

"Four score and seven years ago our fathers brought forth on this continent a new nation, conceived in liberty, and dedicated to the proposition that all men are created equal."

A 'score' means a group or set of 20, so 'four score and seven' is 87.

In the French number system, 80 is 'quatre-vingt' or 'four twenties' and 90 is 'quatre-vingt-dix' or 'four twenties and ten'.

Objective C: Communicating
iii. move between different forms of mathematical representation

Different number bases and symbols are different forms of representation. In this Exploration, show that you can move between different symbols and number bases.

ATL

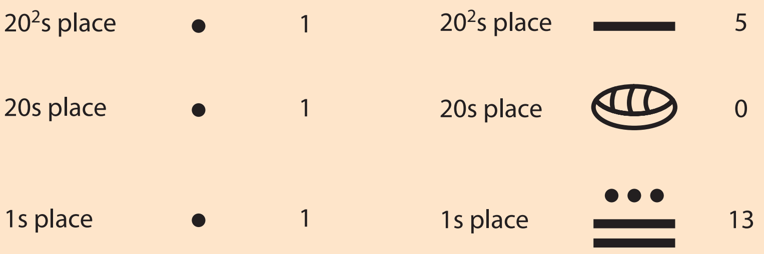
Exploration 3

The Mayans had a base 20 number system, but only three symbols:

They used the symbols ▬ and ● in combination to create numbers up to 19. For example:

1 Write the numbers 4, 10, 14 and 19 in Mayan numerals.

The numbers from 0–19 fulfil the same purpose in Mayan numerals as the numbers from 0–9 do in base 10. Effectively, they are the Mayan digits. For place value, they wrote digits vertically in ascending order. It was important to leave enough space between the digits so that they could tell the difference between, say, a 10 (two horizontal bars) and 105 (two horizontal bars with one in the units place and one in the twenties place).



$(1 \times 20^2) + (1 \times 20) + 1 = 421$

$(5 \times 20^2) + (0 \times 20) + 13 = 2013$

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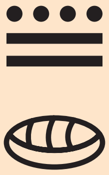
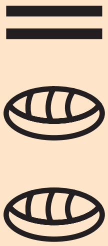
2 Find the value of these Mayan numerals:

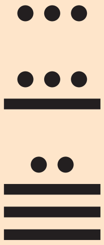
a



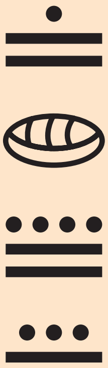
b



c



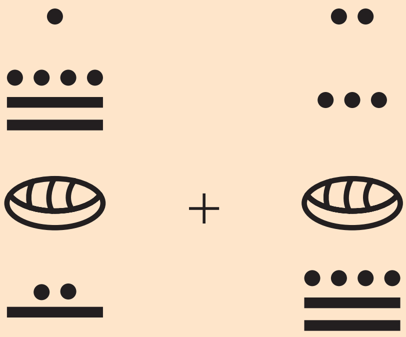
d



3 Write these numbers in Mayan:

a 806
b 2005
c 10 145
d 16 125
e 43 487
f 562 677
g 100 000
h 2 million

4 Find the value of this sum, showing all your steps:



Express your answer in Mayan notation as well as in base 10.

5 Explain why the Mayan system is part additive and part place value.

6 Compare our base 10 number system with the Mayan system and write down any advantages or disadvantages that one has over the other.

The Mayan people kept stingless bees for honey. Technically, stingless bees do have stingers but they are so small that a sting from one is only about as painful as a mosquito bite.

C Performing operations

- How are mathematical operations in other bases similar to and different from operations in base 10?

In a calculation like this:

$$\begin{array}{r} 892 \\ +347 \\ \hline \end{array}$$

you add the units, then the tens, then the hundreds. When the addition gives you a number greater than or equal to 10 (the base) you write down the units digit and ‘carry’ the 10 to the next column.

You can add numbers in other bases in the same way as decimal numbers, ‘carrying’ when a number is greater than or equal to the base number.

Example 4

Find the value of $465_7 + 326_7$.

$$\begin{array}{r} 4\,6\,5 \\ +\,3\,2\,6 \\ \hline {}^{1}\;4 \end{array}$$

Units column: $5 + 6 = 11_{10}$. In base 7 this is 1 seven and 4 units, or 14_7. Write 4 in the units column and carry a 1 into the sevens column.

$$\begin{array}{r} 4\,6\,5 \\ +\,3\,2\,6 \\ \hline {}^{1}\,2\,4 \end{array}$$

Sevens column: $6 + 2 + 1 = 9_{10} = 12_7$.

$$\begin{array}{r} 4\,6\,5 \\ +\,3\,2\,6 \\ \hline 1\,1\,2\,4 \end{array}$$

7^2 column: $4 + 3 + 1 = 8_{10} = 11_7$.

So, $465_7 + 326_7 = 1124_7$

Reflect and discuss 4

You could work out $465_7 + 326_7$ by adding, as in Example 4, or by:

- converting 465_7 and 326_7 into base 10
- adding
- converting the result back to base 7.

Which method would you prefer? Explain.

For subtraction, you can ‘borrow’ the base number, in the same way that you ‘borrow’ a 10 in base 10.

Example 5

Calculate $783_9 - 267_9$.

$$\begin{array}{r} 7\,8\,3 \\ -\,2\,6\,7 \\ \hline \end{array}$$

$7 > 3$, so you cannot subtract 7 from 3.
Rewrite 7 8 3 as 7 7 ${}^{1}3$, because 3 units, 8 nines and 7 eighty-ones is the same as 13 units, 7 nines and 7 eighty-ones.

$$\begin{array}{r} 7\,7\,{}^{1}3 \\ -\,2\,6\;\;7 \\ \hline 5 \end{array}$$

Find $13_9 - 7_9$ by writing it in base 10:
$13_9 - 7_9 = 12_{10} - 7_{10} = 5_{10} = 5_9$

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$$\begin{array}{r} 7\,7\,{}^{1}3 \\ -\,2\,6\,\,7 \\ \hline 1\,\,5 \end{array}$$

7 − 6 = 1 in base 9 and in base 10.

$$\begin{array}{r} 7\,7\,{}^{1}3 \\ -\,2\,6\,\,7 \\ \hline 5\,1\,\,5 \end{array}$$

7 − 2 = 5

So, $783_9 - 267_9 = 515_9$

Practice 4

1 Calculate:

a $124_8 + 321_8$ **b** $77_8 + 261_8$ **c** $563_8 + 241_8 + 757_8$

2 Calculate:

a $453_6 - 231_6$ **b** $341_6 - 153_6$ **c** $1231_6 - 402_6$

3 Calculate:

a $115_8 - 23_8 + 45_8$ **b** $2231_7 - 125_7 - 216_7$ **c** $8463_9 + 728_9 - 541_9 + 18_9$

4 Calculate:

a $92A1_{12} + 4436_{12}$ **b** $10\,000_{12} - 123_{12}$

Problem solving

5 Vorbelar the alien does not count in base 10, but in order to make things easy for us to understand, she uses our base 10 symbols in the usual order. She calculates 216 + 165 and obtains the answer 403. Find the value of 216 – 165, giving your answer using Vorbelar's base.

> If you think dogs can't count, try putting three dog biscuits in your pocket and then giving Fido only two of them.
>
> – *Phil Pastoret*

Reflect and discuss 5

When performing a calculation, is it important that the two numbers being added or subtracted are in the same base? Why or why not?

Exploration 4

There are many ways of setting out the calculation $742_{10} \times 36_{10}$.

Here is one approach.

$\begin{array}{r} 7\,4\,2 \\ \times\;\;3\,6 \\ \hline {}^{1}\,2 \end{array}$	First multiply the top row by 6, one column at a time, starting with the 2 in the units column. 6 × 2 = 12, so write 2 in the units column and 1 in the tens column.

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742 × 36 ^{2}52	6 × 4 tens = 24 tens, so add 4 to the 1 in the tens column and write 2 in the hundreds column.
742 × 36 4452	6 × 7 hundreds = 42 hundreds, so add 2 to the 2 in the hundreds column and write 4 in the thousands column. This completes 742 × 6.
742 × 36 4452 ^{1}260	Now calculate 742 × 3 tens. 3 tens × 2 = 6 tens, so write 6 in the tens column. 3 tens × 4 tens = 12 hundreds, so write 2 in the hundreds column and 1 in the thousands.
742 × 36 4452 22260	3 tens × 7 hundreds = 21 thousands, so add 1 to the 1 in the thousands column and write 2 in the ten thousands column. This completes 742 × 3 tens.
742 × 36 4452 + 22260 26712	Add together the two parts already found.

1 Use the method above to calculate $727_{10} \times 736_{10}$. Explain how you have used times tables.

2 Try to use the same method to calculate $727_9 \times 736_9$. Describe any difficulties.

3 Here is a multiplication table in base 9.
Use the table to help you calculate $727_9 \times 736_9$.

×	1	2	3	4	5	6	7	8	10
1	1	2	3	4	5	6	7	8	10
2	2	4	6	8	11	13	15	17	20
3	3	6	10	13	16	20	23	26	30
4	4	8	13	17	22	26	31	35	40
5	5	11	16	22	27	33	38	44	50
6	6	13	20	26	33	40	46	53	60
7	7	15	23	31	38	46	54	62	70
8	8	17	26	35	44	53	62	71	80
10	10	20	30	40	50	60	70	80	100

4 Explain why you need this table for long multiplication in base 9.

Practice 5

1 Here is a multiplication table in base 5:

×	1	2	3	4	10
1	1	2	3	4	10
2	2	4	11	13	20
3	3	11	14	22	30
4	4	13	22	31	40
10	10	20	30	40	100

Use it to complete these multiplications:

a $123_5 \times 24_5$ **b** $401_5 \times 133_5$ **c** $213_5 \times 1310_5$

2 Copy and complete this multiplication table for base 6.

×	1	2	3	4	5	10
1	1	2		4	5	10
2	2	4		12		20
3						
4	4	12				40
5	5					
10	10	20		40		

Hence complete these multiplications:

a $155_6 \times 23_6$ **b** $212_6 \times 315_6$

Problem solving

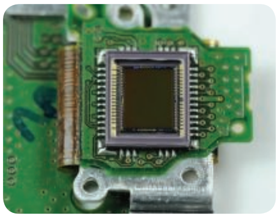
3 Find two numbers (both greater than 1) whose product is 1215_6.

Because long multiplication relies on knowing multiplication table facts, multiplication in different bases is quite tricky. You can write out a multiplication table for the base you need, but in most cases it is easier to convert the numbers to base 10, perform the multiplication, and then convert the numbers back to the required base.

Division also relies on knowing multiplication table facts, so it is usually easiest to convert numbers to base 10 and then divide them.

D A peek at binary numbers

- Would you be better off counting in base 2?
- How does form influence function?



Computers use base 2 (binary) for most of their operations. One reason for this is that the electrical signals that pass through the computer chips are either ‘on’ or ‘off’. As the computer’s memory exists in two states (on or off) it uses just two symbols for its counting system: 0 and 1.

Reflect and discuss 6

- Create a multiplication table in binary. Can you 'learn your times tables' in the new base?
- Try using it to perform some multiplications in base 2.
- Can you work out 1 011 100 110 × 110 110 100 in base 2?

You may find multiplication slow in base 2 because there are a lot of digits, and long multiplication involves a lot of steps. There are fewer symbols in numbers with small bases, and thus numbers have more digits. In numbers with larger bases, there are more symbols, and so numbers have fewer digits.

Hexadecimal is used in computers because each symbol in hexadecimal represents a string of 4 bits, or 4 binary digits. For example, 3F in hexadecimal converts to 0011 1111 in binary. That is why built-in passwords (in modems and routers for example) often use digits 0-9 and letters A-F.

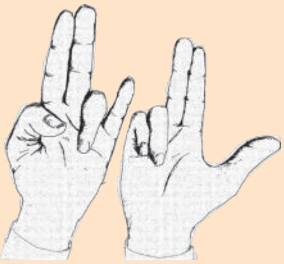
Exploration 5

These hand diagrams show a way of counting in binary using your fingers. An extended finger/thumb represents 1 and a curled one represents 0.

This diagram represents zero.

This diagram represents one.

1 Find the number in base 10 represented by the diagrams below. It may help you to find the number in binary first.



2 Draw finger diagrams to show each of these base 10 numbers in binary:

a 32_{10} **b** 68_{10} **c** 101_{10} **d** 250_{10}

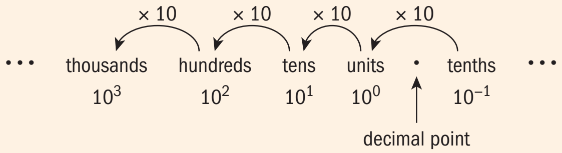
Reflect and discuss 7

- Is counting in base 2 on your fingers a more efficient way of counting?
- How is counting in base 2 on your fingers more difficult than counting in base 10?
- When might it be useful to try to count in base 2 on your fingers?

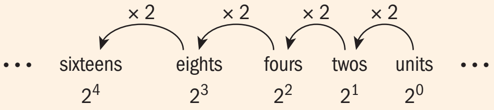
Summary

Our number system, often called Arabic numerals, is a **place value system.** The position of a digit tells you its value.

Our number system is also known as decimal, or base ten, or denary, because the value of each place value column is 10 times the value of the column to its right.



You can make a place value system with any natural number base. In binary, or base two, each column is worth twice the column to its right.



The subscript 2 means that the number $10\,111_2$ is written in base 2.

The base of a number system tells you how many unique symbols the number system uses. Base 10 has ten unique number symbols: 0 through 9. Base 2 has two unique symbols: 0 and 1.

When you write numbers in bases greater than 10, letters are used for the extra symbols needed.

In base 12 the symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A and B (or sometimes lowercase letters a and b).

You can add numbers in other bases in the same way as decimal numbers, ‘carrying’ when a number is greater than or equal to the base number.

For division and multiplication in a different base, it is usually most efficient to convert the numbers to base 10, do the calculations, then convert the answer back to the base in question.

Mixed practice

1 **Find** the value in base 10 of:

a 1101_2
b $110\,110_2$
c $1\,000\,101_2$
d 1221_3
e $12\,000_3$
f 4523_8
g $34\,337_8$
h $18A9_{11}$
i $4EA8_{16}$

2 **Find** the value in the base specified of:

a 100_{10} in base 2
b 100_{10} in base 3
c 100_{10} in base 4
d 255_{10} in base 2
e 1008_{10} in base 3
f 8162_{10} in base 5
g 271_{10} in base 12
h 456_{10} in base 12
i 1689_{10} in base 12
j 2785_{10} in base 12
k 1049_{10} in base 16
l $15\,342_{10}$ in base 16

3 You can measure angles in degrees, minutes and seconds. One degree = 60 minutes or $60'$, and one minute = 60 seconds or $60''$. $45^\circ\,15'\,30'' =$ 45 degrees, 15 minutes and 30 seconds.

Convert these angle measures:

a 42.8° into degrees, minutes and seconds
b 90.15° into degrees, minutes and seconds
c 2.52° into degrees, minutes and seconds
d $25^\circ 15'$ into base 10 (including a decimal)
e $62^\circ\,30'\,30''$ into base 10 (including a decimal)
f $58^\circ\,12'\,20''$ into base 10 (including a decimal)

4 **Calculate**, giving your answers in the base used in the question:

a $1101_2 + 1001_2$
b $11\,101_2 + 101\,101_2$
c $1011_2 + 10\,111_2 + 1\,101\,101_2$

d $122_3 + 2101_3$ **e** $2122_3 + 20\,012_3$

f $3012_4 + 10\,332_4$

g $33\,010_4 + 312\,212_4 + 101_4$

h $565_{12} + 718_{12}$ **i** $1A91_{12} + 3302_{12}$

j $AB0_{12} + 10B8_{12}$ **k** $6E_{16} + AC_{16}$

l $F00D_{16} + FACE_{16}$

5 **Calculate**, giving your answers in the base used in the question:

a $11\,011_2 - 101_2$ **b** $111\,011_2 - 11\,101_2$

c $10\,110_2 - 111_2$ **d** $2210_3 - 102_3$

e $3011_4 - 231_4$ **f** $44\,125_6 - 5150_6$

g $50\,112_6 - 1045_6$ **h** $B791_{12} - 89A0_{12}$

i $FEED_{16} - BEEF_{16}$

6 If the current time was 10:45, **use** addition and subtraction to **find** the time that is:

a 35 minutes from now

b $2\frac{1}{2}$ hours from now

c 3 hours and 40 minutes from now

d 1 hour and 19 minutes ago

e 6 hours and 50 minutes ago

7 **Calculate**, giving your answers in the base used in the question.

Either create multiplication tables like the ones in Practice 5, or convert the numbers to base 10 and then convert back.

a $110_2 \times 10_2$ **b** $1011_2 \times 11_2$ **c** $1101_2 \times 101_2$

d $2101_3 \times 21_3$ **e** $14_5 \times 23_5$ **f** $24_5 \times 131_5$

8 A set of weights contains weights in grams that are powers of 4. There are three of each size weight in the set. The lightest weight is 1 g, and the heaviest is 16 384 g.

a **Find** the total weight of these six weights: two of the smallest weights, three of the next size and one of the size above that.

b **Write down** the weights you would use to create a total weight of 26 g. Remember that there are only three of each type of weight.

c **Find** the value of 2313_4 in base 10. **Explain** why this would be relevant if you wanted to find the total weight of three 1 g weights, one of the next size, three of the next size and two of the size after that.

Problem solving

d Ronnie has $331\,213_4$ grams in weights. Donnie has $231\,232_4$ grams in weights. Connie wishes to weigh out the same amount as Ronnie and Donnie combined, using the weights from her sets. Remembering that she has only three of each type of weight, **determine** the weights Connie should use.

e Ted weighs out 1330_4 grams. Ed weighs out twice that amount. Ned weighs out twice as much as Ed and Ted combined. Remembering that he has only three of each type of weight, **determine** the weights Ned should use.

9 A congressional committee decides to streamline the US monetary system. All coins will be phased out except for 1 cent, 5 cents and 25 cents. All existing bills (bank notes) will be phased out and replaced with bills valued \$1, \$5, \$25, \$125 and \$625.

a A mathematician suggests that the dollar should be revalued to be worth 125 cents, not 100 cents as it is currently.

i **Suggest** reasons why the mathematician thinks this might be a good idea.

ii **Suggest** reasons why it might not be a good idea.

b **Find** 540_{10} in base 5. **Hence find** the smallest number of notes that you would need to give somebody \$540 under the new system.

c **Find** 732_{10} in base 5 and 246_{10} in base 5.

d Rania owes \$732 to her credit card company. She makes a payment of \$246. **Find** the total amount outstanding (still owed), and the least number of notes she would need to repay the amount.

e In real life, monetary systems do not use regular bases, but have combinations of notes that follow irregular patterns. For example, in the UK, there are £1 and £2 coins, then notes valued £5, £10, £20 and £50. **Suggest** reasons why it would not be convenient to use a perfectly regular base as suggested above.

Review in context

ATL

In the 1960s and 1970s, NASA's Pioneer and Voyager missions launched spacecraft to travel beyond the solar system. Each contained messages in case the craft was intercepted by extra-terrestrial life forms. Scientists tried to choose things that they hoped could be universally communicated, or which could be measured by aliens.

1 If an alien culture has a number system, there is a good chance that they will have some sort of place value system. They will probably have an idea of whole numbers, and maybe an awareness of prime numbers. One suggestion for a message to include might be to communicate the first prime numbers in binary. The first five numbers in binary are: 10, 11, 101, 111 and 1011.

- **a** **Write down** the values of these numbers in denary (decimal).
- **b** **List** the next ten prime numbers in denary.
- **c** **Hence list** the first 15 prime numbers in binary.

Problem solving

2 For the messages, the scientists did not use 1s and 0s; these symbols are known only to humans. Instead, they used horizontal and vertical lines: — and | .
Here are some sequences given in binary using the symbols — and | .
Determine whether — or | represents 1. Justify your answer.
Describe each sequence and **predict**, using — and | , the next three terms.

- **a** |, |——, |——|, |————, | |——|
- **b** |, |, |—, | |, |—|, |———, | |—|, |—|—|
- **c** |, | |, |——|, | |—| |, |—|———|

3 The scientists also encoded some information about our solar system. They used information about quantities that can be counted, rather than measured, because measurements require units whereas counting does not. One quantity that can be counted is the number of 'days' in each planet's year. Earth has (roughly) 365 Earth days per year because that is how many times it rotates around its own axis while orbiting the sun. The table gives some information about the planets in our solar system:

Planet	Year length	Day length
Mercury	87.96 Earth days	1408 Earth hours
Venus	224.68 Earth days	5832 Earth hours
Earth	365.26 Earth days	24 Earth hours
Mars	686.98 Earth days	25 Earth hours
Jupiter	11.862 Earth years	10 Earth hours
Saturn	29.456 Earth years	11 Earth hours
Uranus	84.07 Earth years	17 Earth hours
Neptune	164.81 Earth years	16 Earth hours

- **a** **Calculate** the number of Earth hours in a Mars year. Give your answer in denary.
- **b** **Hence find** the number of Mars days in a Mars year. Give your answer in denary.
- **c** **Find** (to the nearest whole number) the number of Mars days in a Mars year in binary.
- **d** **Find**, in binary, the number of days in a year for each of the other outer planets (Jupiter to Neptune).
- **e** **Describe** any problems you would encounter when trying to perform a similar calculation for Venus.